Algebra II 6.5 Properties of Logarithms

Key

Obj: Use the properties of logs to rewrite equations

Fill in the table with the corresponding properties of exponents.

Properties of Exponents		
$a^m \cdot a^n = a^{m+n}$	$\frac{a^m}{a^n} = a^{m-n}$	$(a^m)^n = a^{mn}$

take note

Properties Properties of Logarithms

For any positive numbers m, n, and b where $b \neq 1$, the following properties apply.

Product Property $\log_b mn = \log_b m + \log_b n$

Quotient Property $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property $\log_b m^n = n \log_b m$

Use the properties of logarithms to complete each equation.

$\log 20 = \log(5 \cdot 4) = \log 5 + \log 4$

$\log_5 (x^9) = 9 \log_5 x$

$\log_2 24 - \log_2 3 = \log_2 \frac{24}{3} = \log_2 8$

$\log_3(5x^4) = \log_3 5 + \log_3 x^4 = \log_3 5 + 4 \log_3 x$

Example 2: Expanding Logarithms. (separate all products or quotients first)

a. $\log_5 a^2 b^7$

$\log_5 a^2 + \log_5 b^7$ prod.

$2\log_5 a + 7\log_5 b$ power

b. $\ln\frac{25}{3} = \ln 25 - \ln 3$ Quot.

You try.

a. $\log_7 \frac{r^3 t^4}{v}$

$\log_7 r^3 + \log_7 t^4 - \log_7 v$

$3\log_7 r + 4\log_7 t - \log_7 v$

b. $\log\frac{7}{225}$

$\log 7 - \log 225$

c. $\log_7 \frac{x^2}{3v}$

$\log_7 x^2 - \log 3v$

$2\log_7 x - (\log 3 + \log v)$

$2\log_7 x - \log 3 - \log v$

any thing in denom. will be subtracted if separat...

Example 3: Simplifying Logarithms

What is each expression written as a single logarithm? Simplify the single logarithm if possible. **Hint:** Bring in coefficients as exponents first.

write log only once in answer

a. $4\log_4 m + 3\log_4 n - \log_4 p$

$\log_4 m^4 + \log_4 n^3 - \log_4 p$

$\log_4 \frac{m^4 n^3}{p}$

b. $3\ln 2 - 2\ln 5$

$\ln 2^3 - \ln 5^2$

$\ln 8 - \ln 25$

$\ln \frac{8}{25}$

You try.

a. $5\log_2 c - 7\log_2 n - \log_2 p$

$\log_2 c^5 - \log_2 n^7 - \log_2 p$

$\log_2 \frac{c^5 n^7}{p}$

b. $2\ln 7 + \ln 2$

$\ln 7^2 \cdot 2$

$\ln 98$

Use the properties of logs to solve an unknown.

Given $\log_3 5 = 1.4650$ *and* $\log_3 20 = 2.7268$, Find

a. $\log_3 100$

write 100 as product, power or quotient of givens

$\log_3 5 \cdot 20$

$\log_3 5 + \log_3 20$

$1.4650 + 2.7268$

4.1918

b. $\log_3 4$

$\log_3 \frac{20}{5}$

$\log_3 20 - \log_3 5$

$2.7268 - 1.4650$

1.2618

c. $\log_3 125$

$\log_3 5^3$

$3\log_3 5$

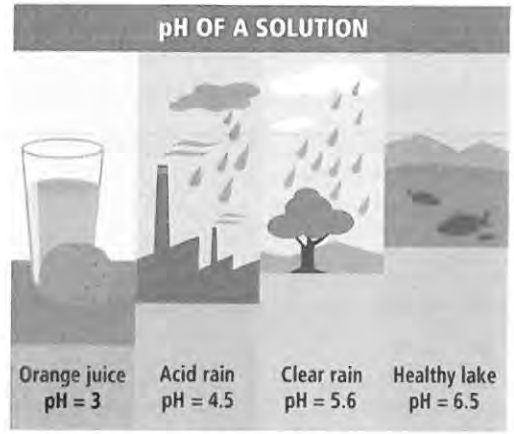
$3(1.4650)$

4.3950

Example 4. Apply properties of logs.

The pH of a solution is a measure of its concentration of hydrogen ions. This concentration (measured in moles per liter) is written [H⁺] and is given by the formula $pH = \log\frac{1}{[H^+]}$

What is the concentration of hydrogen ions in the acid rainfall?



$4.5 = \log\frac{1}{H^+}$

$4.5 = \log 1 - \log H^+$ → 0

$-4.5 = \log H^+$ $10^{-4.5} = H^+$

What is the concentration of ions in a liter of OJ?

$3 = \log\frac{1}{H^+}$

$3 = \cancel{\log 1} - \log H^+$

$-3 = \log_{10} H^+$ $10^{-3} = H^+$

Find the approximate value of any log

Change of base formula
$\log_b m =$ log m/log b or ln m/ln b

you can use any base but ln + log are the most common

Example 5: Using the Change of Base Formula

a. $\log_2 3$ $\frac{\log 3}{\log 2} \approx 1.585$

or $\frac{\ln 3}{\ln 2} \approx 1.585$

b. $\log_2 7$ $\frac{\log 7}{\log 2} \approx$

Example 6. Use change of base to solve b^x equations. use ln or log.

a. $2^x = 7$

rewrite

$\log 2^x = \log 7$

$x \log 2 = \log 7$ power rule

$x = \frac{\log 7}{\log 2} =$

b. $3^x = 15$

$\ln 3^x = \ln 15$

$x \ln 3 = \ln 15$

$x = \frac{\ln 15}{\ln 3}$